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ABSTRACT

We compute the three-loop corrections to the quark axial vector form factor in massless QCD, focusing on the pure-singlet contributions where the axial vector current couples to a closed quark loop. Employing the Larin prescription for γ_5, we discuss the UV renormalization of the form factor. The infrared singularity structure of the resulting singlet axial-vector form factor is explained from infrared factorization, defining a finite remainder function.

The quark form factors describe the coupling of a quark-antiquark pair to an external current, which can be a vector, scalar, axial-vector or pseudo-scalar. Higher order perturbative corrections to these form factors provide important universal information [1–3] on anomalous dimensions, and they constitute the purely virtual corrections to important collider processes such as gauge boson production, Higgs boson decay or deeply inelastic scattering. After renormalization of ultraviolet (UV) divergences, the form factors remain divergent due to infrared (IR) poles.

For massless quarks, the three-loop QCD corrections to vector [4–6], scalar [7] and pseudo-scalar [8] form factors were derived in the literature, and important progress has been made recently towards the four-loop corrections to the vector form factor [9–12]. All massive form factors are known to two-loop order in QCD [13–16], supplemented by partial three-loop results [17].

Owing to chirality conservation, the massless vector and axial-vector form factors can differ from each other only through contributions where the external current couples to a closed quark loop, which is then connected to the external quark-antiquark pair through virtual gluon exchanges. These so-called pure-singlet contributions (PS) occur for the first time at two loops, where they vanish for the vector form factor while yielding an IR-finite contribution for the axial-vector form factor [15]. The three-loop pure-singlet contributions to the vector form factor were computed earlier [5,6] and found to be IR-finite. All contributions where the axial vector current insertion couples to the external quarks are denoted as non-singlet (NS), and the sum of non-singlet and pure singlet contributions yields the singlet (S) form factor. In the present letter, we derive the three-loop pure singlet contributions to the axial-vector form factor in massless QCD, thereby completing the full set of massless three-loop quark form factors.

Using dimensional regularization to handle both infrared and ultraviolet divergences, one must extend the four-dimensional chirality projection operator γ_5 to symbolic $d = 4-2\epsilon$ space-time dimensions. For this purpose, we follow the prescription introduced by Larin [18], which replaces the symmetrized axial vector vertex factor as follows:

$$\gamma_\mu\gamma_5 \to \frac{1}{2}\left(\gamma_\mu\gamma_5 - \gamma_5\gamma_\mu\right) \to \frac{i}{6}\epsilon_{\mu\nu_1\nu_2\nu_3}\gamma^{\nu_1}\gamma^{\nu_2}\gamma^{\nu_3}\,. \tag{1}$$

It is based on the original γ_5 formulation of t'Hooft and Veltman [19], which was further refined by Breitenlohner and Maison [20], but has the further advantage that the Lorentz index space does not need to be split into 4-dimensional and $(d-4)$-dimensional subspaces, thereby allowing all Lorentz algebra to be performed in d dimensions throughout. In the Larin scheme, a finite renormalization of the axial vector current is required (besides the conventional UV-renormalization) to restore chirality conservation of massless quarks and to ensure the validity of the chiral anomaly. The axial renormalization constants in the Larin scheme were known to three-loop order for the non-singlet contributions for a long time [18,21], while the finite contribution to the renormalization of the singlet current has been derived only most recently [22].

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The axial vector vertex function is obtained by the insertion of an off-shell axial-vector current with virtuality $q^2 = (p_1 + p_2)^2$ between a quark-antiquark pair with on-shell momenta p_1 and p_2, yielding the Born-level expression

$$\bar{u}(p_2)\Gamma^{\mu}_{A,0}u(p_1) = \bar{u}(p_2)\gamma^{\mu}\gamma_5 u(p_1)\,. \tag{2}$$

The axial vector form factor is then obtained by applying a projection operator on the all-order vertex function Γ^{μ}_{A}:

$$\mathcal{F}_A = -\frac{3}{4(1+\epsilon)(1-\epsilon)(1-2\epsilon)(3-2\epsilon)q^2}\,\mathrm{Tr}\left(\not{p}_2\,\Gamma^{\mu}_{A}\not{p}_1\,\gamma_{\mu}\gamma_5\right)\,, \tag{3}$$

where the Larin prescription (1) is to be applied throughout. With the above normalization, the Born-level axial vector form factor is equal to unity: $\mathcal{F}^{(0)}_A = 1$. The amplitude-level calculations closely follow those of the three-loop vector form factor [6]. The bare form factor is expanded in the bare QCD coupling constant $a_B = \alpha_{s,B}/(4\pi)$ as follows:

$$\mathcal{F}^B_A = \sum_{i=0}^{\infty} a^i_B \mathcal{F}^{B,(i)}_A\,. \tag{4}$$

The $\overline{\mathrm{MS}}$ renormalization of the axial vector vertex function Γ^{μ}_{A} involves [18,21,22] the renormalization of the coupling constant Z_g and of the axial vector current insertion $Z^{ms}_5 Z^f_5$, where Z^{ms}_5 and Z^f_5 denote the divergent and finite parts of the axial vector renormalization constant. They depend on the prescription used for γ_5 in dimensional regularization. Different axial vector renormalization constants are required for the singlet and non-singlet axial vector form factors. They have been computed to three-loop order in the Larin scheme for the non-singlet [18,21] and singlet [22] axial vector current. Their expansion in the renormalized QCD coupling constant $a = \alpha_s(\mu^2)/(4\pi)$ reads:

$$Z = \sum_{i=0}^{\infty} a^i Z^{(i)}\,. \tag{5}$$

After renormalization, the massless non-singlet form factors for axial vector and vector agree with each other in their finite parts that are obtained by subtracting their universal infrared pole structure [4,23–25], as required by chirality conservation for massless fermions and as obtained for a naively anti-commuting γ_5. To extract the pure singlet axial vector form factor, one takes the difference of the renormalized singlet and non-singlet axial vector form factors:

$$\mathcal{F}_{A,\mathrm{PS}} = \mathcal{F}_{A,\mathrm{S}} - \mathcal{F}_{A,\mathrm{NS}}\,. \tag{6}$$

By taking the difference of the singlet and non-singlet renormalization constants [18,21,22], we define pure-singlet renormalization constants which turn out to be useful in arranging [15] the different contributions in terms of non-singlet and pure-singlet type:

$$\begin{aligned} Z^{ms}_{5,\mathrm{PS}} &= Z^{ms}_{5,\mathrm{S}} - Z^{ms}_{5,\mathrm{NS}} \\ &= C_F\,N_{F,J}\left(\frac{3}{\epsilon}a^2 + \frac{(-66+109\epsilon)C_A - 54\epsilon C_F + (12+2\epsilon)N_F}{9\epsilon^2}a^3\right) + \mathcal{O}(a^4)\,, \end{aligned} \tag{7}$$

$$\begin{aligned} Z^{f}_{5,\mathrm{PS}} &= Z^{f}_{5,\mathrm{S}} - Z^{f}_{5,\mathrm{NS}} \\ &= C_F\,N_{F,J}\left(\frac{3}{2}a^2 + \frac{(-326+1404\zeta_3)C_A + (621-1296\zeta_3)C_F + 176N_F}{54}a^3\right) + \mathcal{O}(a^4)\,, \end{aligned} \tag{8}$$

where $C_A = N$, $C_F = (N^2-1)/(2N)$ are the QCD colour factors. The overall power of $N_{F,J}$ can be identified with the number of quark flavours that couple to the external axial vector current, while N_F is the number of massless quark flavours. In the following, we take $N_{F,J} = 1$ throughout.

Expanding out the pure singlet axial vector form factor (6) in powers of the renormalized QCD coupling constant,

$$\mathcal{F}_{A,\mathrm{PS}} = a^2\mathcal{F}^{(2)}_{A,\mathrm{PS}} + a^3\mathcal{F}^{(3)}_{A,\mathrm{PS}} + \mathcal{O}(a^4)\,, \tag{9}$$

one finds the following expressions at two and three loops:

$$\mathcal{F}^{(2)}_{A,\mathrm{PS}} = \mathcal{F}^{B,(2)}_{A,\mathrm{PS}} + Z^{ms,(2)}_{5,\mathrm{PS}} + Z^{f,(2)}_{5,\mathrm{PS}}\,, \tag{10}$$

$$\mathcal{F}^{(3)}_{A,\mathrm{PS}} = \mathcal{F}^{B,(3)}_{A,\mathrm{PS}} + \left(Z^{ms,(1)}_{5,\mathrm{NS}} - \frac{2\beta_0}{\epsilon}\right)\mathcal{F}^{B,(2)}_{A,\mathrm{PS}} + \left(Z^{ms,(2)}_{5,\mathrm{PS}} + Z^{f,(2)}_{5,\mathrm{PS}}\right)\mathcal{F}^{B,(1)}_{A,\mathrm{NS}} + Z^{f,(1)}_{5,\mathrm{NS}}Z^{ms,(2)}_{5,\mathrm{PS}} + Z^{ms,(3)}_{5,\mathrm{PS}} + Z^{f,(3)}_{5,\mathrm{PS}}\,, \tag{11}$$

where $\beta_0 = (11C_A/3 - 2N_F/3)$. All bare form factors are computed using the Larin prescription throughout.

The two-loop contribution has been computed previously. We reproduce the result of [15] and provide higher order terms in the ϵ expansion, as required for the study of the infrared singularity structure at higher loop orders:

$$\begin{aligned} \mathcal{F}^{(2)}_{A,\mathrm{PS}} = C_F\Bigg[&-18 + 6L_\mu + \frac{2\pi^2}{3} + \epsilon\left(-\frac{345}{4} + 3L_\mu + 6L^2_\mu + 4\zeta_3 + \frac{59\pi^2}{18}\right) \\ &+\epsilon^2\left(-\frac{2579}{8} + 3L^2_\mu + 4L^3_\mu + \frac{146}{3}\zeta_3 + \frac{1469\pi^2}{108} + \frac{4\pi^4}{45}\right) + \mathcal{O}(\epsilon^3)\Bigg]\,, \end{aligned} \tag{12}$$

where we have introduced $L_\mu = \log(-q^2/\mu^2)$. It should be noted that only the finite term in the above expression is independent on the prescription used for γ_5, while all ϵ-type terms are specific to the Larin scheme.

The three-loop pure singlet axial vector form factor is our main new result. It reads

$$\begin{aligned}\mathcal{F}^{(3)}_{A,\mathrm{PS}} = &\frac{1}{\epsilon^2} C_F^2 \left(36 - 12L_\mu - \frac{4\pi^2}{3}\right) + \frac{1}{\epsilon} C_F^2 \left(\frac{453}{2} - 24L_\mu - 12L_\mu^2 - 8\zeta_3 - \frac{77\pi^2}{9}\right)\\ &+ C_F^2 \left(1108 - 75L_\mu + \pi^2 L_\mu - 24L_\mu^2 - 8L_\mu^3 - \frac{328}{3}\zeta_3 - \frac{1225\pi^2}{27} - \frac{3\pi^4}{5}\right)\\ &+ C_F N_F \left(\frac{469}{9} - \frac{76}{3}L_\mu + \frac{8\pi^2}{9}L_\mu + 4L_\mu^2 - \frac{40\pi^2}{27}\right)\\ &+ C_F C_A \left(-\frac{7403}{18} + \frac{538}{3}L_\mu - \frac{44\pi^2}{9}L_\mu - 22L_\mu^2 + 62\zeta_3 + \frac{385\pi^2}{27} - \frac{17\pi^4}{90}\right).\end{aligned} \tag{13}$$

The divergent contributions in the above expression are of infrared origin. They can be expressed by applying the one-loop infrared singularity operator [23]

$$\mathrm{I}^{(1)}_{q\bar{q}} = -C_F \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon}\right)\left(1 - \epsilon^2 \frac{\pi^2}{12}\right), \tag{14}$$

such that

$$\mathcal{F}^{(3,\mathrm{finite})}_{A,\mathrm{PS}} = \mathcal{F}^{(3)}_{A,\mathrm{PS}} - \mathrm{I}^{(1)}_{q\bar{q}} \mathcal{F}^{(2)}_{A,\mathrm{PS}} \tag{15}$$

$$\begin{aligned} = &C_F^2 \left(\frac{409}{2} - 66L_\mu - \frac{16\pi^2}{3} - \frac{8\pi^4}{15}\right)\\ &+ C_F N_F \left(\frac{469}{9} - \frac{76}{3}L_\mu + \frac{8\pi^2}{9}L_\mu + 4L_\mu^2 - \frac{40\pi^2}{27}\right)\\ &+ C_F C_A \left(-\frac{7403}{18} + \frac{538}{3}L_\mu - \frac{44\pi^2}{9}L_\mu - 22L_\mu^2 + 62\zeta_3 + \frac{385\pi^2}{27} - \frac{17\pi^4}{90}\right)\end{aligned} \tag{16}$$

is finite. The finite remainder functions that are obtained from infrared pole subtraction [23–25] were previously observed [26] to be independent on the prescription that is being used to define internal and external polarization states in dimensional regularization. It can therefore be expected that (16) is independent on the γ_5-scheme, while (13) is valid only in the Larin scheme.

We observe that the finite pure-singlet contribution of the three-loop vector form factor [5,6] contains terms proportional to ζ_5, which are not present in (13) or (16). The finite parts of the three-loop non-singlet form factors even contain π^6, which are absent in their pure-singlet counterparts. This remarkable lowering of transcendental weight deserves further study.

The three-loop pure-singlet axial vector form factor contributes to coefficient functions for observables in polarized hadron collisions and to the three-loop coefficient function for hadronic Z-boson production. In the latter, its contribution cancels for mass-degenerate quark isospin doublets in the loop, such that a sizable effect can be expected only from the third-generation quarks. For these, the massless form factor computed here provides a reliable description of the bottom quark contribution, while the top quark contribution remains to be derived.

In this letter, we completed the calculation of the three-loop quark form factors in massless QCD by deriving the pure-singlet axial vector form factor at this order. Our computation employed the Larin scheme for γ_5 throughout in all amplitudes and projectors. The three-loop pure-singlet axial vector form factor is infrared-divergent. Its singularity structure in accordance with the expectation from infrared factorization, such that a finite remainder function can be defined.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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